Pitch Control in Harmonica Playing

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ABSTRACT: The technique used by harmonica players to alter the pitch of the note being played, by vocal tract manipulations, is described. Observations of the effect from the player's point of view, and the results of experiments using a mechanically blown instrument are presented. An acoustical analysis of the effect using the small signal approximation, and including both reeds in each airway in the model, yields predictions in accord with the observations.

INTRODUCTION

The purpose of this report is to describe, and explain, the technique which is used by harmonica players (particularly blues and jazz players) to alter the pitch of the note being played by changing the shape of the vocal tract, particularly by changes in the position of the tongue.

There are only casual references to the harmonica in the literature, usually in general discussions of the vibrating reed as a sound source, but the acoustics of the instrument do not appear to have been studied in depth. However, there is an extensive literature of the mechanism of sound generation by vibrating reeds [1], [2], [3], [4], [10], [11], [17], particularly in connection with the clarinet. Also there is a body of evidence for the effect of vocal tract resonances on the performance of wind instruments [5], [6], [7], [8], [9], including their pitch [7], [12].

The instrument on which the technique is most widely used is the simple ten hole harmonica, which is tuned to a diatonic major scale. The more complex chromatic harmonica does not readily respond to the technique, and is much less widely used in this field despite the apparent advantage of a full chromatic scale. We have used Hohner "Special 20 Marine Band" harmonicas, which are a general standard. They are available in any major key.

For a C instrument the tuning is as shown in Figure 1. The missing A4 in the lowest octave is to allow the dominant 7th chord to be played without dissonance.

The "classical" technique of playing the instrument is to cover four holes with the lips and to block the lowest three off with the tongue. The melody is played through the remaining open hole and the tongue can be lifted to allow vamping of accompanying chords. The tongue is not available to alter the shape of the vocal tract, and pitch bending is not used with this method. With this technique, the instrument is very limited because accidentals are not available.

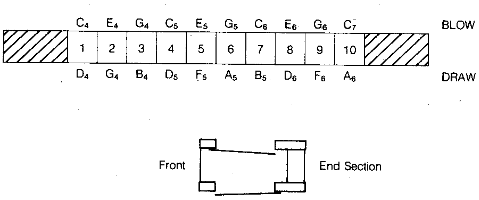

Figure 1: Tuning and reed layout of the ten hole harmonica

The instrument was made much more versatile by the adoption of a different technique by American Negroes, earlier this century. The method involves "kissing" the harmonica to select the note to draw or blow. The tongue is then free to be used to change the shape of the mouth cavity which has the effect of changing the pitch in a remarkably subtle and reliable fashion (changes of up to three semitones can be achieved).

The importance of being able to bend pitch for this type of music is twofold. Firstly, Jazz/Blues music uses a lot of subtle slides of pitch, rather than fixed pitch scale tones. Any instrument that cannot produce these "bent notes" is of little use for the idiom. Secondly, the scales used are not diatonic major scales. They require flattened thirds, fifths and sevenths to be available. This can be achieved by playing modes of the major scale (particularly that with tonic a fourth below the instrument key), and other missing notes are played by bending available ones.

OBSERVATIONS ABOUT PITCH BENDING

Any theory of pitch bending on the harmonica must account for the following observations.

A. The note can only be flattened.

B. Only certain notes can be bent — low draw notes and high blow notes. The detailed rule is simple — the only notes that can be bent are those where the other note in the same channel (i.e. the draw note when a blow note is being played) has a lower pitch than the one being played. For the harmonica shown in Figure 1, this applies to draw 1 to 6 and blow 7 to 10. Blow 1 to 6 and draw 7 to 10 cannot be bent more than a few tens of cents.

C. The degree to which the pitch can be bent is also related to the pitch of the other note in the same channel. The rule is that, for those notes that can be bent, the pitch can be varied from the normal pitch of the note being played, down to approximately a semitone sharp of the pitch of the other note in the same channel (which is flat of the note being played by A).

D. For draw notes the pitch variation is essentially continuous between the upper and lower pitch limits for a continuous change in mouth geometry. For the high notes, the pitch change tends to be abrupt between the limits.

E. The technique that is used to achieve these changes, while complex, is essentially as follows. For medium to high pitched notes the size of the oral cavity, controlled by the position of the tongue, seems to be the crucial factor. For medium pitch draw bends, the tongue is pushed down and back to flatten the pitch. For the high blow bends, the tongue is pushed forward and as mentioned above the pitch drops more or less abruptly. In both cases the higher notes are played with the tongue further forward in the mouth. For very low pitched notes the movement of the tongue is less pronounced and it is noticed that the Adam's apple drops on bending to lower pitch, and this is an indication that the larynx is being lowered [5]. These changes in tongue position from low to high notes are similar to those found in woodwind playing [5], [6].

OBSERVATIONS USING AN ARTIFICIALLY BLOWN HARMONICA

Experiments were performed using a mechanically blown harmonica, to show that the pitch bending effect could be produced by varying the resonance frequency of a chamber through which the instrument was blown. The arrangement is depicted schematically in Figure 2. The effect of changes in the vocal tract geometry was simulated by a variable length cylinder in the air supply. This arrangement has also been used experimentally by Coltman [12]. The frequency was measured with a Cohn Strobe Tuner, and the measurements are taken close to the critical pressure where the small signal approximation is most likely to apply (there is a small flattening of pitch with increasing blowing pressure).

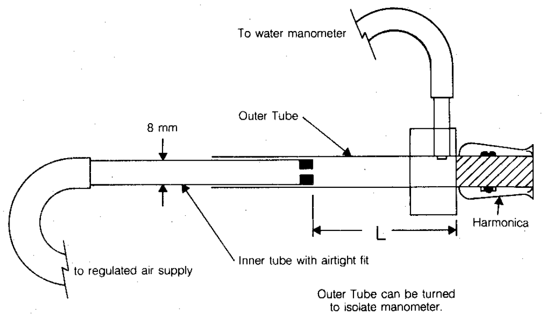

Figure 2: Schematic layout of experimental apparatus

Figure 3 shows the results of an experiment in which the frequency of the sound emitted by blow 8 on a C instrument, a note that bends easily, is measured as the tube was extended. The free reed frequency of the bottom plate reed (i.e. the one not normally associated with the production of the note being played) is then returned sharper, that is towards the pitch of the top plate reed by filing the reed. The pitch variation with cylinder length is re-measured and the process repeated. The results show clearly that the limit down to which the pitch can be bent is determined by the free reed frequency of the other reed in the channel. It is also observed that this reed has a substantial amplitude of vibration when the pitch is lowest. The extremes of pitch that can be produced by this experimental arrangement are in good agreement with what is found playing the instrument normally.

Figure 4 shows the pitch variation with cylinder length, for the note blow 8 on a G instrument, with both reeds in the channel free to vibrate, with the top reed free to vibrate with the bottom reed taped over, and with the bottom reed free to vibrate with the top reed taped over. The results show that the vibration of both reeds is needed if the greatest pitch variation is to be obtained.

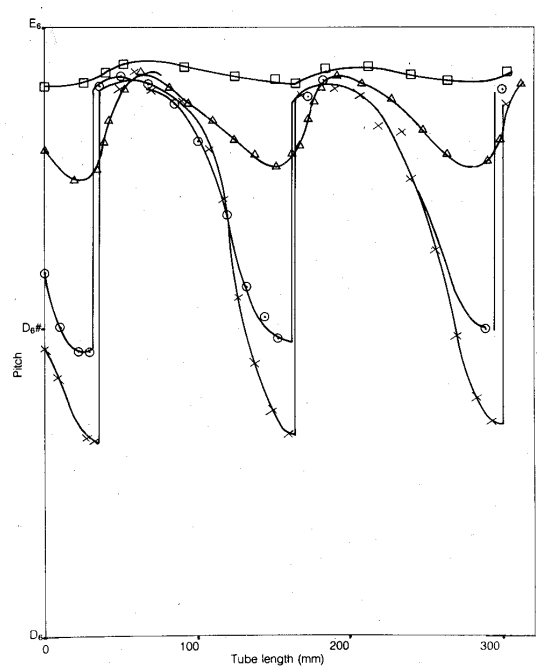

Figure 3: Pitch versus tube length for a blow note that will bend (blow 8 on a C instrument), the bottom plate (draw) reed being tuned to different free reed pitches, approaching the free reed pitch of the top plate.

× — D6 + 30 cents *△ — D6 + 25 cents*
⊙ — D6 — 44 cents *□ — E6 — 25 cents*

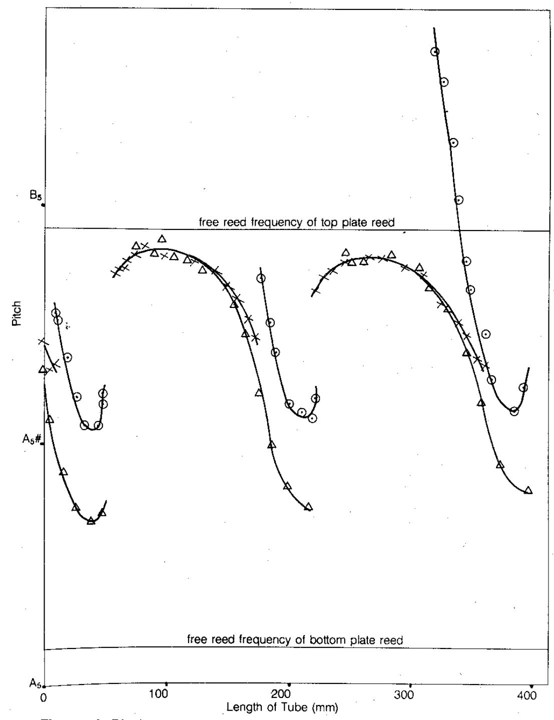

Figure 4: Pitch versus tube length for a blow note that will bend (blow 8 on a G instrument).

△ — Both reeds free to vibrate *× — Only top plate reed free*
⊙ — Only bottom plate reed free

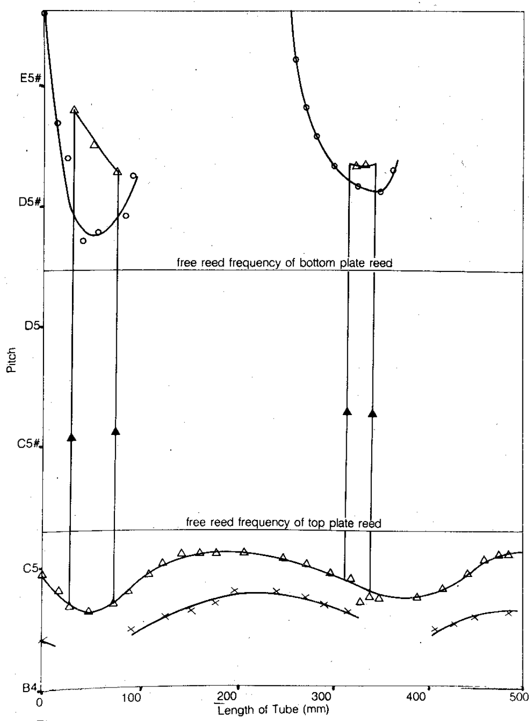

Figure 5: Pitch versus tube length for a blow note that will not bend significantly (blow 4 on a C instrument).
△ — Both reeds free to vibrate ⊙ — Only top plate reed free
× — Only bottom plate reed free

Figure 5 shows the same measurements repeated for a note that will not bend easily — blow 4 on a C instrument. The pitch variation is small, and, with both reeds free, at cylinder lengths where the pitch is flattened, the sound output greatly attenuated. In these length ranges, it is also possible, by increasing the blowing pressure, to produce a second pitch near the pitch produced by blowing with only the top reed free. Such a note can be produced in normal playing by delicate vocal tract manipulation and increased blowing pressure.

Measurements have been made of the critical pressure required to start vibration. For the "bendable" note "blow 8" on a G instrument, for instance, it was found to range from 0.1 kPa to 0.5 kPa for the top reed only, 4.6 kPa to 6 kPa for the bottom reed, and from 0.3 kPa to 1.5 kPa when both reeds are free to vibrate. In this last case, the high value occurs for tube lengths that yield the minimum pitch, that is, the critical pressure increases as the note is bent flat.

That the effect can be produced with such an arrangement, is strong evidence that pitch bending in normal playing is affected by changing the resonant frequency of the vocal tract by changing its shape. That there is an increase in threshold pressure as the note is bent, leads many players to falsely ascribe the bending effect to increased blowing pressure, or to choking the air supply. Furthermore, our experiments show that both reeds in the channel are involved in pitch bending, contrary to the general belief that only one reed is vibrating at any time.

THEORY

Fletcher [3] has shown that the oscillations of the classical wind instruments can be understood by dividing the instrument system into a passive linear distributed acoustic system (the instrument tube), and a non-linear sound generator. Each system is characterised by its impedance (or admittance) function, and owing to the possibility of negative impedance of the sound generator, self sustained oscillations can occur.

This approach can be used to give a qualitative explanation of pitch control in harmonica playing. The passive distributed system is the player's vocal tract with admittance Y_v, at the lips looking into the mouth, and the sound generator is the pair of reeds of the harmonica in the airway of the note being played, with admittance Y_h, looking into the instrument. The condition that the reed system act as a sound generator is that the real part of Y_h be negative, and larger in magnitude than the real part Y_v [3]. In addition, continuity of the volume velocity requires that

$$\tan \phi_h = \tan \phi_v \qquad (1)$$

where ϕ_v and ϕ_h are the phases of the admittance functions of the vocal tract and the reed system. Equation (1) determines the frequency of oscillation of the combined system. Details of the calculation of Y_h, ϕ_h, Y_v and ϕ_v are given in the Appendix.

It is helpful to distinguish, following Helmholtz [1], two ways in which a reed that is coupled to a distributed linear acoustic system can act as a sound generator. In one mode the reed gap is reduced when the reed moves in the direction of the air flow and in the other it is increased when it moves in the direction of the air flow. We shall call the first a closing reed and the second an opening reed. (We have deviated from Helmholtz's terminology because it is only appropriate to blown instruments, not drawn ones.) When playing a blow note the top reed is a closing reed and the bottom reed is an opening reed. The roles

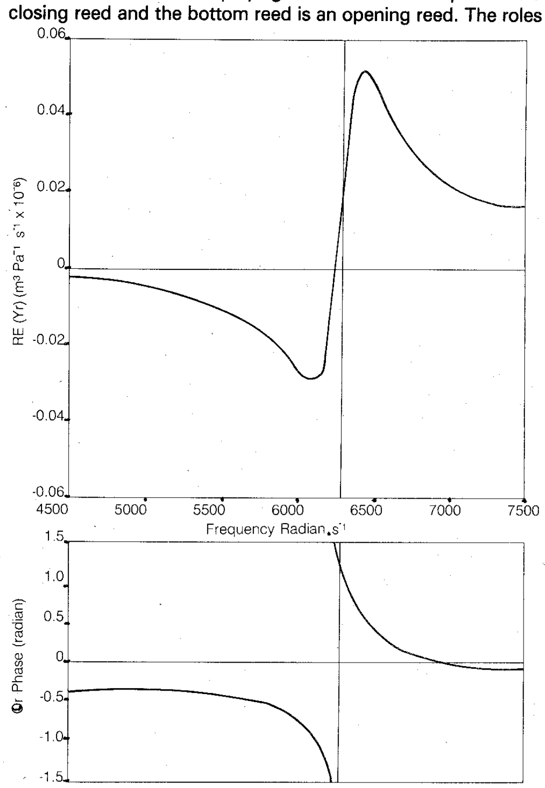

Figure 6: The real part, and the phase of the acoustic admittance versus frequency for a closing reed.

$X_u = 0.2\,mm$	$b = 2.00\,mm$
$M_r = 0.01\,g$	$a = 1.0\,mm$
$S_r = 0.2\,cm^2$	$D_r = 0.05$
$W_r = 6300\,rad.s^{-1}$	$P_o = 1.0\,kPa$
$y = 1$	$z = 0.5$

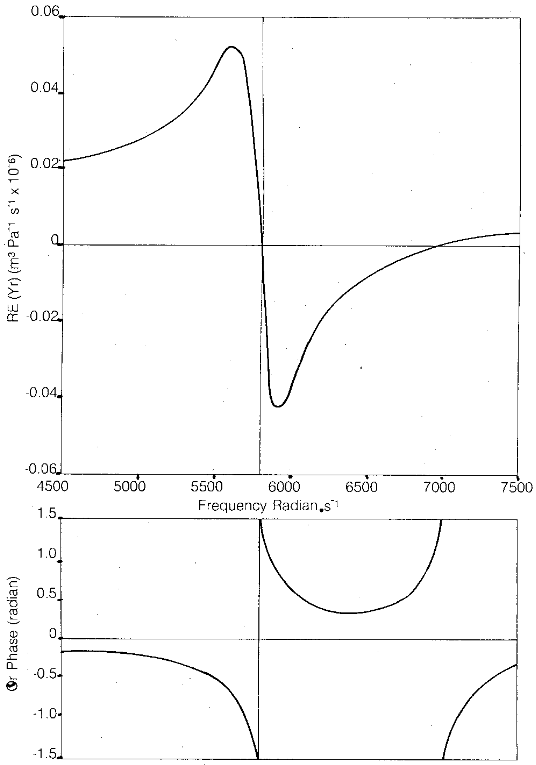

Figure 7: The real part, and the phase of the acoustic admittance versus frequency for an opening reed. Parameters are as for Figure 6 except $X_u = -0.2$ mm, $W_r = 5800$ rad. s^{-1}

are reversed for a draw note. The reeds that are associated with the normal playing of notes are the closing reeds, but the following analysis will show that the opening reed plays a decisive role in bent notes.

Consider first the case where only one reed in the airway is free to vibrate. Figure 6 and Figure 7 show the real part of the admittance Y_r and the phase ϕ_r for opening and closing reeds. This form of the acoustic admittance of reeds has been well confirmed experimentally [4]. Operation near the minimum of $RE(Y_r)$ is favoured by the system, and this occurs for closing reeds at a frequency just below the reed's resonance frequency, and for opening reeds at a frequency just above the reed's resonance frequency, as was found in Figures 4 and 5. The frequency of vibration of the combined system of vocal tract and one reed with be given by

$$\tan \phi_r = \tan \phi_v \qquad (2)$$

For closing reeds, $-\pi/2 < \phi_r < 0$ for the frequencies where $RE(Y_r) < 0$ and for opening reeds $0 < \phi_r < \pi/2$ for the frequencies where $RE\ (Y_r) < 0$.

Since ϕ_v varies from nearly $+\pi/2$ to $-\pi/2$, with suitable continuous variations in the geometry of the vocal tract, equation (2) can only be satisfied for certain vocal tract shapes for closing reeds, and for certain intermediate vocal tract shapes for opening reeds. It is in fact found that when one reed of a harmonica is covered by tape, the instrument will only sound when certain mouth shapes are assumed. This was also found for the artificially blown instrument with one reed fixed in Figures 4 and 5.

In the real instrument both reeds, which are acoustically in parallel, contribute to sound generation. One reed is opening and the other is closing, so there are two distinct cases:

1. The higher pitch reed is a closing reed and the lower is opening. This applies to HIGH BLOW and LOW DRAW.

2. The higher pitch reed is an opening reed and the lower is closing. This applies to LOW BLOW and HIGH DRAW.

In view of observation B above, it seems that only the first of these cases allows pitch bending. We can see why this is by plotting the admittance given by equations (A7) and (A8) for the two distinct cases. This is done is Figures 8 and 9.

We find that for the first case (closing reed of higher pitch), Y_h is negative essentially only between the two reed resonance frequencies. Furthermore, the acoustic phase angle varies from $+\pi/2$ to $-\pi/2$ in the range where $RE(Y_h)$ is negative when $W_b < W_t$. Since ϕ_v also varies virtually from $+\pi/2$ to $-\pi/2$ with changing vocal tract shape, it will be possible for the instrument to sound at any pitch between the resonance ferquencies of the reeds for some mouth geometry. This is the pitch bending phenomenon.

For the second case (opening reed of higher pitch), by contrast, we find the frequency ranges in which $RE(Y_h)$ is negative are essentially those for the individual reeds, and, in these regions, the phase angle only asumes a small range of values. Thus, sound is possible only for fairly specific mouth geometries at two small frequency ranges; below the closing reed's resonance and above the opening reed's resonance. The first is the frequency of the normal note and the other is the note that can be produced by "overblowing" a low blow note. This latter note can be struck on the low blow notes by applying the same technique for bending low draw notes and

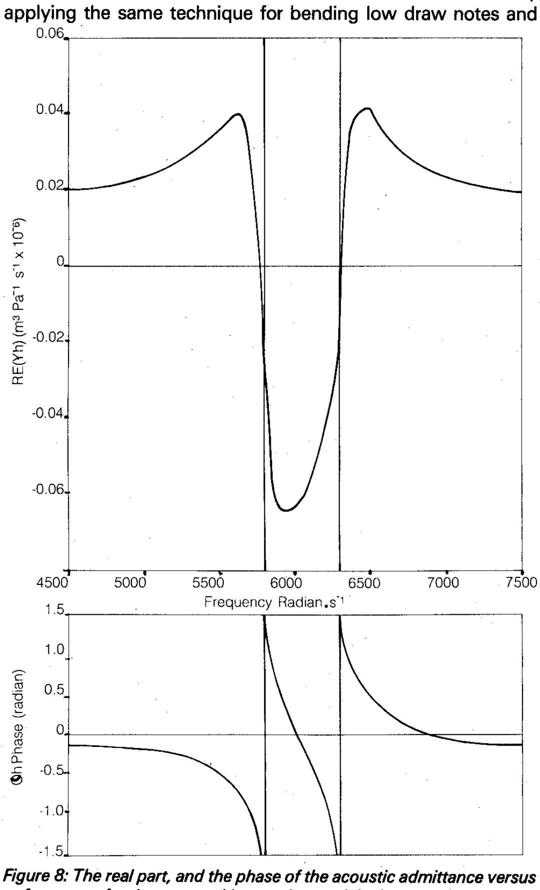

Figure 8: The real part, and the phase of the acoustic admittance versus frequency for the two reed harmonica model when the resonance frequency of the closing reed is higher than the resonance frequency of the opening reed. All parameters are as in Figure 6 except $W_t = 6300$ rad. s^{-1}, $W_b = 5800$ rad. s^{-1}

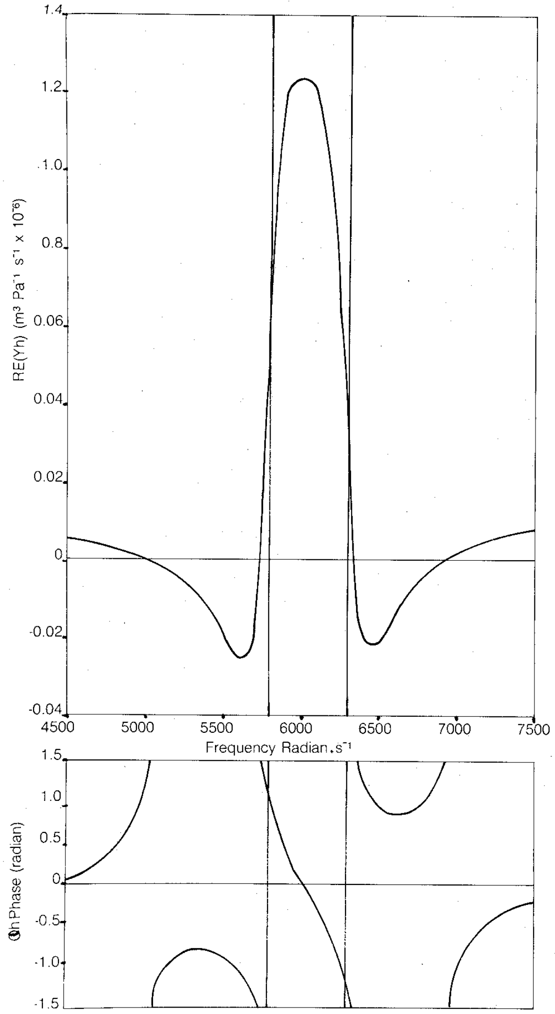

Figure 9: The real part, and the phase of the acoustic admittance versus frequency for the two reed harmonica model when the resonance frequency of the closing reed is lower than the resonance frequency of the opening reed. All parameters are as in Figure 6 except W_t = 5800 rad. s^{-1}, W_b = 6300 rad. s^{-1}, P_o = 2kPa.

blowing very hard. It is very difficult to control and often forms a rapid alternation of pitch with the normal note. This is occasionally used by players as an effect.

Observation C can be understood in terms of our model, when it is realised that it expresses the lowest pitch attainable in terms of the pitch of the other note in the channel when played normally i.e. as a closing reed. Our model suggests that the lowest attainable pitch will be about that of the other reed in the channel played as an opening reed. The minimum of $RE(Y_r)$ for an opening reed occurs at approximately 0.5 D_rW_r above the reed resonance frequency, and the same amount below the resonance frequency for closing reeds [3]. We therefore expect the lowest extreme of pitch to be about D_rW_r above the pitch of the other note of the same channel played normally. This is in agreement with the fixed scale interval relation expressed in C.

The observations D and E are explained by reference to the variation of pitch with tube length shown in Figure 3. When operating in the region of continuous variation in pitch with length, increasing the length of the tube lowers the pitch. Evidently this is the type of change in vocal tract resonance occurring in the technique for the low and middle notes. However the pitch can also be lowered by crossing the discontinuity in pitch with a decrease in length of the pipe. This accounts for the apparently contradictory technique used on high blow notes, and their discontinuous change in pitch.

AERODYNAMIC DAMPING OF THE REED

Observation A is clearly related to the observation that the closing reed requires much less pressure to start vibration than the opening reed in the same channel, and thus the note associated with the closing reeds is taken as the natural pitch of the note. In all notes that can be bent, the closing reed has the higher pitch. If the closing reeds were not easier to sound the instrument would be practically unplayable, requiring constant attention to pitch control. However, this large difference in critical pressure of opening and closing reeds of almost identical parameters is not predicted by the simple model above.

It can be shown from equation (A3) that the minimum critical pressure P_{min} is approximately

$$P_{min} = (2M_rz/S_ry)X_oD_r \qquad (3)$$

and the difference in X_o or M_r/S_r is not sufficient to account for the large observed difference in P_{min}. Only the assumption that the opening and closing reeds have substantially different values for the damping constant D_r will make (3) accord with reality.

The need for different values of D_r for the opening and closing reed is also suggested by other observations. Firstly, as mentioned above, the simple model predicts that the operating frequency of the closing and opening reeds should differ from the reed resonance by a ratio proportional to the internal damping of the reed, whereas we find (Figure 4) that the closing reed operates much closer to the resonance frequency than the opening reed. Secondly the simple model predicts that when the two reeds are tuned to the same fequency $RE(Y_h)$ is always positive so the instrument cannot sound. This is contrary to observation, but the problem is avoided if damping factors are set unequal.

The source of this difference in damping would appear to be the aerodynamic mechanism described by St Hilaire et al [17]. By an analysis of the time varying potential flow around an harmonium reed, they found that terms in the time-dependent Bernouli equation could give rise to an oscillating force on the reed that is in phase with the reed velocity if the reed is a closing reed and of opposite phase to the velocity if the reed was an opening reed. The reeds of a harmonium are always arranged to be closing, and they considered that the aerodynamic mechanism was the cause of the excitation of the harmonium reed. That the opening reeds can be excited while playing the harmonica indicates that the interaction of the reeds with the vocal tract resonances is the most important mechanism for reed excitation in this case, but the aerodynamic force on the reed, being in phase with the velocity of the reed for closing reeds, can be viewed as decreasing the damping of the reed, and being out of phase with the reed velocity for opening reeds, can be viewed as an additional damping mechanism.

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APPENDIX—
The vocal tract and harmonica admittance functions

THE VOCAL TRACT

The resonances of the vocal tract have been extensively studied by workers in the field of speed synthesis [13], [14]. In vocalisation, the glottis is the sound source, and the sub-gottal system is generally ignored. The system from the glottis to the lips is then represented by a series of cylinders of varying area, and linear acoustics are used to calculate its response. In reed instrument playing, the sound source is at the lips and in general the glottis is wide open, so it is not so easy to justify ignoring the sub-glottal system.

In principle, we could use the published methods [15] to calculate the complex admittance function of the vocal tract as seen from the lips, given data on the area variation of the vocal tract, glottis and sub-glottal system. Since this data does not exist for harmonica playing this approach is not practical. Fortunately we can make progress in understanding pitch bending, with only the most qualitative knowledge of the variation with frequency of the phase of the admittance of the vocal tract.

To this end we start by considering a tube of uniform cross sectional area S, and length L, open at the far end. The input impedance can be put in the form [3]

$$Z_p = (R_oC/S)\,(1 + jH\tan(KL))\,/\,(H + j\tan(KL)) \qquad (A1)$$

where R_o is the density of air, C the speed of sound and K the wave number. H is the height of the impedance maxima above the reference level R_oC/S. The admittance is $1/Z_p$. The phase of the admittance, that is, the phase of the flow into the pipe, relative to the pressure in the pipe, ϕ_p, is thus given by

$$\tan(\phi_p) = -[(H^2 - 1)/2H]\sin(2KL). \qquad (A2)$$

Since H is typically 10—100 for a tube of these dimensions, ϕ_p varies from nearly $-\pi/2$ to $+\pi/2$ as frequency is increased through a resonance, with the change being most rapid near a resonance. (See ref. [16] for experimental measurements of acoustic phase for a straight tube and for various horns.) The same type of variation of phase with frequency near resonance is found for tubes of non-uniform area also [3], [16] although the resonance frequencies are no longer harmonically related and are related in a complex way to the area variation of the tube. Thus, while the exact relation between resonance frequency and vocal tract geometry remains obscure, we can make the following deduction; for a given frequency of operation, the phase of the acoustic admittance of the vocal tract seen from the lips, can be varied from nearly $+\pi/2$ to $-\pi/2$, by suitable continuous variations in the geometry of the vocal tract (mainly affected by changes in tongue position) that alter the relationship of the vocal tract resonances to the operating frequency.

THE HARMONICA

On the basis of a small signal model of the vibrating reed first introduced by Backus [2], Fletcher [3] has given expressions for admittance of reeds near the threshold blowing pressure.

The admittance of the reed Y_r, seen from the passive system, is given by

$$Y_r = (1-B)/[(|P_o|/zU_o)\cos\phi_r - W(R_oa/b|X_o|)\sin\phi_r] \qquad (A3)$$

where

P_o = the blowing pressure in the mouth referred to atmospheric
U_o = the magnitude of the constant part of the volume flow
W = the frequency of vibration
W_r = the resonance frequency of the reed
a = the length of the mass of air bounded by the reed opening
b = the width of the reed
X_u = the unblown displacement of the reed toward the inside of the instrument
X_o = the equilibrium displacement of the reed toward the inside of the instrument
$X_o = X_u - (S_r/M_rW_r^2)P_o$
M_r = the effective mass of the reed
S_r = the effective area of the reed
y,z = reed parameters that relate to the pressure/volume relation that is assumed in the model
D_r = the reed damping coefficient
$A = [S_r(P_oy/zX_o)WW_rD_r]/\{M_r[(W_r^2 - W^2) + (D_rWW_r)^2]\}$
$B = [S_r(P_oy/zX_o)(W_r^2 - W^2)]/\{M_r[(W_r^2 - W^2) + (D_rWW_r)^2]\}$

and the phase, ϕ_r, of the admittance is

$$\phi_r = \frac{(|P_o|/zU_o)A + W[R_o(a/b)|X_o|(B-1)]}{(|P_o|/zU_o)(1-B) + W[R_o(a/b)|X_o|]A} \qquad (A4)$$

Equations (A3) and (A4) are generalisations of Fletcher's results to allow positive or negative pressures in the pipe. When P_o/X_o is positive, the reed is a closing reed and, when P_o/X_o is negative, it is opening.

The real and imaginary parts of Y_r are

$$RE(Y_r) = Y_r\cos\phi_r \qquad (A5)$$

$$IM(Y_r) = Y_r\sin\phi_r \qquad (A6)$$

and the condition that RE (Y_r) be negative is essentially that $(1-B)$ be negative.

For a model of the harmonica in which both the opening and the closing reed contribute to the generation of sound, we take the volume flow velocity in the channel to be the complex sum of the velocities through the two reeds. Since the pressure acting on both reeds is essentially the same, (that is the two reed generators are acoustically in parallel) we can write

$$RE[Y_h(W)] = RE[Y_r(W_t,W,X_o)] + RE[Y_r(W_b,W,-X_o)] \qquad (A7)$$

$$IM[Y_h(W) = IM[Y_r(W_t,W,X_o)] + IM[Y_r(W_b,W,-X_o)] \qquad (A8)$$

$$\tan\phi_h = IM[Y_r(W)]\,/\,RE[Y_r(W)] \qquad (A9)$$

where

Y_h = the admittance of the two reed harmonica seen from the mouth
ϕ_h = the acoustic phase of the two reed harmonica
W_t = the resonant frequency of the top plate reed
W_b = the resonant frequency of the bottom plate reed

We have assumed in equations (A7) and (A8) that the physical parameters of both the reeds are the same and that they only differ in the sign of the equilibrium opening and in their resonance frequency. This is only a computational aid, and in any case it is a good approximation since the reeds in the same channel, sounding at nearby pitches, are very similar in dimensions.

There are apparently four cases to consider:

1. P_o positive and $W_t < W_b$. This applies to blow notes 1—6. (LOW BLOW)
2. P_o positive and $W_t > W_b$. This applies to blow notes 7—10. (HIGH BLOW)
3. P_o negative and $W_t < W_b$. This applies to draw notes 1—6. (LOW DRAW)
4. P_o negative and $W_t > W_b$. This applies to draw notes 7—10. HIGH DRAW)

However, because of the invariance of equation (A3) under the simultaneous change of sign of P_o and X_o, if we use the same pair of resonance frequencies, case 1 and 4 would yield an identical admittance function as would cases 2 and 3. That is to say, there are only the two distinct cases considered in the text.